# VECTORS

**1** Show that the line joining D(2,2,3) and E(4,3,2) is parallel to the line joining F(5,3,–2) and G(9,5,–4).

**2** Show that A(2,5,0), B(5,8,3) and C(4,7,2) are collinear and find the ratio AB:BC.

**3** P divides the line joining S(1,0,2) and T(5,4,10) in the ratio 1:3. Find the coordinates of P.

**4** Use the **scalar product** to prove that the triangle with vertices P(1,0,0), Q(1,1,1) and R(0,1,1) is right-angled.

**5** A go-kart driver is being affected by two forces modelled by the vectors:

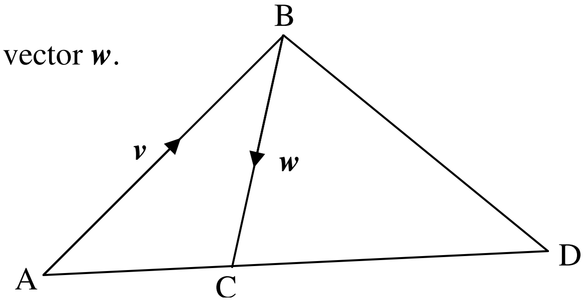
$$\boldsymbol{u} = \begin{pmatrix} 8 \\ 7 \\ -3 \end{pmatrix} \text{ and } \boldsymbol{v} = \begin{pmatrix} 9 \\ -4 \\ 5 \end{pmatrix}$$

(a) Calculate the resultant force.

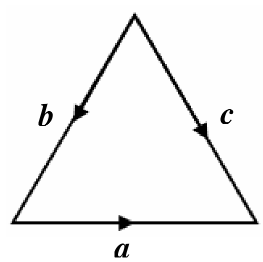
(b) Calculate the magnitude of the resultant force.

(c) Calculate the acute angle between the two forces.

**6** $\overrightarrow{AB}$ represents vector $\boldsymbol{v}$ and $\overrightarrow{BC}$ represents vector $\boldsymbol{w}$.
$AC{:}CD = 1{:}3$.
Find vector $\overrightarrow{BD}$ in terms of $\boldsymbol{v}$ and $\boldsymbol{w}$.




**7** The sides of this equilateral triangle are 2 units long and represent the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ as shown in the diagram.
Evaluate $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$.



1. $D(2,2,3) \quad E(4,3,2) \quad F(5,3,-2) \quad G(9,5,-4)$

$$\vec{DE} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \qquad \vec{FG} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$$

$$\vec{FG} = 2\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$$\vec{FG} = 2\vec{DE} \Rightarrow \vec{DE} \parallel \vec{FG}$$

2. $A(2,5,0) \quad B(5,8,3) \quad C(4,7,2)$

$$\vec{AB} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \qquad \vec{BC} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

$\vec{AB} \parallel \vec{AC}$ & share a common point

$\Rightarrow$ A, B & C are collinear

$$\vec{AB} = 3\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \vec{BC} = -1\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{AB} : \vec{BC} = 3 : -1$$

3.

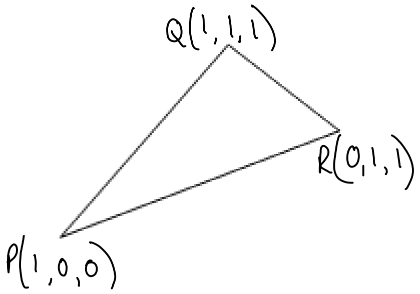

$$\vec{SP} = \frac{1}{4}\vec{ST} \qquad \underline{p} = \underline{s} + \vec{SP}$$

$$\vec{SP} = \frac{1}{4}\begin{pmatrix} 4 \\ 4 \\ 8 \end{pmatrix} \qquad \underline{p} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\vec{SP} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \qquad \underline{p} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

$$\Rightarrow P(2,1,4)$$

4.




$$\vec{PQ} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad \vec{QR} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \qquad \vec{RP} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{PQ} \cdot \vec{QR} = 0.(-1) + 1.0 + 1.0$$

$$= 0$$

$\Rightarrow \vec{PQ} \perp \vec{QR}$

$\Rightarrow \triangle PQR$ is right angled at Q

5. a) $\underline{u} = \begin{pmatrix} 8 \\ 7 \\ -3 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} 9 \\ -4 \\ 5 \end{pmatrix}$

$\underline{u} + \underline{v} = \begin{pmatrix} 17 \\ 3 \\ 2 \end{pmatrix}$

b) $|\underline{u} + \underline{v}| = \sqrt{(17)^2 + (3)^2 + (2)^2}$

$= \sqrt{302}$

c) $\cos\theta = \dfrac{\underline{u}\cdot\underline{v}}{|\underline{u}||\underline{v}|}$

$|\underline{u}| = \sqrt{(8)^2 + (7)^2 + (-3)^2}$

$= \sqrt{122}$

$\cos\theta = \dfrac{8.9 + 7.(-4) + (-3).5}{\sqrt{122}.\sqrt{122}}$

$|\underline{v}| = \sqrt{(9)^2 + (-4)^2 + (5)^2}$

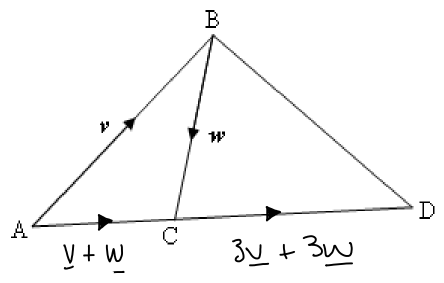
$= \sqrt{122}$

$\cos\theta = \dfrac{29}{122}$

$\theta = \cos^{-1}\left(\dfrac{29}{122}\right)$

Acute angle $= 76.3^\circ$

6.




$\vec{AC} = \underline{v} + \underline{w}$

$\vec{AC} : \vec{CD} = 1 : 3$

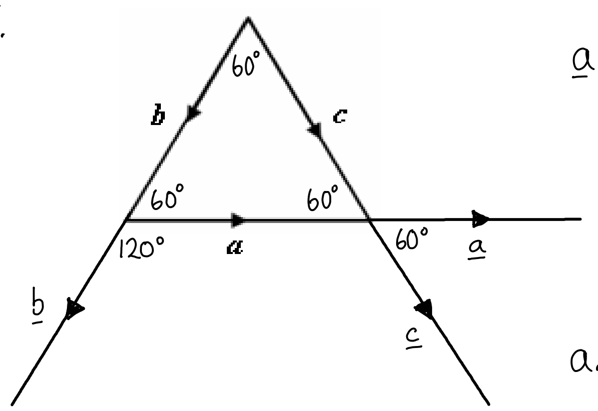
$\Rightarrow \vec{CD} = 3(\underline{v} + \underline{w})$

$= 3\underline{v} + 3\underline{w}$

$\Rightarrow \vec{BD} = \underline{w} + 3\underline{v} + 3\underline{w}$

$= 3\underline{v} + 4\underline{w}$

7.




$a.(\underline{a} + \underline{b} + \underline{c}) = \underline{a}.\underline{a} + \underline{a}.\underline{b} + \underline{a}.\underline{c}$

$= (2)^2 + (2)(2)\cos 120^\circ + (2)(2)\cos 60^\circ$

$= 4 + 4.\left(-\frac{1}{2}\right) + 4.\left(\frac{1}{2}\right)$

$= 4 - 2 + 2$

$a.(a + b + c) = 4$

* (vectors must point away from the angle)